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Deconstruction of Gravity

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We review how one can construct a *deconstructed* gravity by a transverse latticification of 5D General Relativity. The obtained theory is a multigravity theory, with *link fields* that are explicitly constructed out of the metric. We also discuss the spectrum of the theory at the level of the linearized theory.

KEY WORDS: gravity; deconstruction; extra-dimensions; massive gravity.

1. INTRODUCTION

An interesting procedure has recently been proposed to obtain from a five-dimensional Yang–Mills theory a four-dimensional theory which approaches the five-dimensional one in the infrared and which has a finite number of modes (Arkani-Hamed *et al.*, 2001; Hill *et al.*, 2001). The key point in the approach is to replace the extra component of the vector field by bifundamental scalars which can be viewed as arising from the latticized Wilson line along the fifth dimension. A similar construction for gravity would be highly desirable, for many obvious reasons. If one follows the same path as for Yang–Mills theories (Arkani-Hamed *et al.*, 2003; Arkani-Hamed and Schwartz, 2003; Schwartz, 2003,?), the sought for deconstructed gravity would be likely to look like some type of multigravity theory, and indeed those type of theories are known to suffer from various pathologies related to those of the Pauli–Fierz theory for a single massive graviton (Fierz and Pauli, 1939; Boulware and Deser, 1972; Isham *et al.*, 1971; Aragone and Chela-Flores, 1972; Salam and Strathdee, 1977; Isham and Storey, 1978; Chamseddine, 2003; Cutler and Wald, 1987; Wald, 1987; Boulanger *et al.*, 2001; Damour and Kogan, 2002; Damour *et al.*, 2002, 2003; Dolan and Duff, 1984; Aulakh and

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Sahdev, 1985; Nappi and Witten, 1989; Reuter, 1988). In this work we summarize some results obtained elsewhere by Deffayet and Mourad (2004a) (see also Deffayet and Mourad, 2004b) presenting a construction for five-dimensional gravity, analogous to the one done for Yang–Mills theories. We will also discuss the extend to which the consistency of the higher dimensional theory descends to the discretized version.

Let us then first review the deconstruction of gauge theories (Arkani-Hamed *et al.*, 2001; Hill *et al.*, 2001). Consider a 5D non-Abelian gauge field $A = A^a_A t_a\, dx^A \equiv A_\mu\, dx^\mu + A_5 dy$ with gauge group, e.g., $SO(M)$. Under a y-dependent gauge transformation the transformation rules are

$$A' = uAu^{-1} - u\, du^{-1}, \tag{1}$$

where u is an element of $SO(M)$. These reduce to the 4D y-dependent transformations for $A_\mu dx^\mu$, and A_5, which is a scalar viewed from 4D, has the following transformation

$$A_5' = uA_5u^{-1} - u\partial_y u^{-1}, \tag{2}$$

which gives for an infinitesimal transformation

$$\delta A_5 = \partial_y \epsilon - [A_5, \epsilon]. \tag{3}$$

It is very convenient when discretizing the y dimension to replace A_5 by the Wilson line (Kogut, 1983; Wilson, 1974)

$$W(y', y) = Pe^{\int_y^{y'} A_5\, dy}, \tag{4}$$

which transforms linearly as

$$W'(y', y) = u(y')W(y', y)u^{-1}(y). \tag{5}$$

The lattice version of the gauge theory is now easily obtained: one has N sites, on each site a gauge field with a corresponding gauge group $SO(M)_i$ and on each link between neighboring sites a scalar $W(y_i, y_{i+1})$ transforming in the bifundamental of the gauge groups $SO(M)_i \times SO(M)_{i+1}$. One can then write an effective action for the gauge fields and the scalars. The continuum limit is recovered when the number of sites goes to infinity and a goes to zero; the vacuum expectation value of the scalars is then the identity:

$$W(i, i+1) = 1 - aA_5(y) + \cdots, \tag{6}$$

where a is the lattice spacing. In the broken phase one has a massless gauge boson corresponding to the diagonal subgroup and a collection of massive spin one particles with masses

$$m_k = a^{-1} \sin \frac{k\pi}{N}, \qquad k = 1, \ldots N-1. \tag{7}$$

These reproduce, in the infrared, the Kaluza–Klein spectrum of the first modes when the radius is given by aN. So, with this procedure, one is able, from a collection of 4D theories, to get a theory which looks as a 5D theory in the infrared (Arkani-Hamed *et al.*, 2001; Hill *et al.*, 2001).

2. DECONSTRUCTION OF GRAVITY

Let us now turn to gravity, with the aim of reaching the same goal following the same path. For this purpose, we start then from the 5D Einstein–Hilbert Action, rewritten à la ADM (Arnowitt *et al.*, 1962; Wald, 1984) as

$$M_{(5)}^3 \int d^4x\, dy \sqrt{-g}\mathcal{N}\{R + K_{\mu\nu}K_{\alpha\beta}(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta})\}, \tag{8}$$

where $K_{\mu\nu}$ is the extrinsic curvature of surfaces Σ_y:

$$K_{\mu\nu} = \frac{1}{2\mathcal{N}}(g'_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu). \tag{9}$$

Here D_μ is the covariant derivative associated with the induced metric $g_{\mu\nu}$ and a prime denotes an ordinary derivative with respect to y. Note that we chose the surfaces Σ_y to be timelike. We then consider the gauge transformations generated by the vector fields

$$\tilde{\xi} = \xi^A\partial_A = \xi^\mu\partial_\mu + \xi^5\partial_y = \xi + \xi^5\partial_y. \tag{10}$$

They act on the metric as

$$\delta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}, \tag{11}$$

$$\delta\bar{N} = [D_y, \xi] = \partial_y\xi - [\bar{N}, \xi], \tag{12}$$

$$\delta\mathcal{N} = \xi(\mathcal{N}), \tag{13}$$

where we have defined the shift vector by

$$\bar{N} = N^\mu\partial_\mu, \tag{14}$$

and the *covariant* derivative, D_y, by

$$D_y = \partial_y - \bar{N}. \tag{15}$$

Then, under y-dependent four-dimensional diffeomorphisms, that is for $\xi^5 = 0$, we have

$$\delta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}, \tag{16}$$

$$\delta\bar{N} = [D_y, \xi] = \partial_y\xi - [\bar{N}, \xi], \tag{17}$$

$$\delta\mathcal{N} = \xi(\mathcal{N}). \tag{18}$$

The transformation of $g_{\mu\nu}$ and $\mathcal{N}$ is as expected the one of a 4D metric and scalar respectively. The transformation of $\bar{N}$ has however an additional term with respect to the usual one characterizing the transformation of a vector. This new term is reminiscent of the inhomogeneous term contributing to the transformation of a gauge field: the Lie bracket in (17) is replaced in (3) by a matrix commutator. Indeed this analogy justifies the covariant derivative name we gave to D_y: suppose for example that ϕ is a 5D scalar and consider $\partial_y\phi$, it is not a scalar under a diffeomorphism generated by ξ

$$\delta\partial_y\phi = \partial_y\xi(\phi) = \xi(\partial_y\phi) + (\partial_y\xi)(\phi), \tag{19}$$

whereas $D_y\phi$ is indeed a scalar

$$\delta D_y\phi = \delta\partial_y\phi - (\delta\bar{N})(\phi) - \bar{N}(\delta\phi) = \xi(D_y\phi), \tag{20}$$

where we used the transformation rule of $\bar{N}$ (17). Similarly if T is a tensor then $\mathcal{L}_{D_y}T$ is also a tensor under 4D y-dependent diffeomorphisms. One can thus view the role of $\bar{N}$ as rendering possible the formulation of an action invariant under y-dependent 4D diffeomorphisms.

We next consider diffeomorphisms along the fifth dimension. In fact it is more convenient to consider diffeomorphisms generated by D_y, that is $\tilde{\xi} = \zeta D_y$, with ζ depending on y as well as on x. A short calculation gives the following rules

$$\delta g_{\mu\nu} = \zeta\mathcal{L}_{D_y}g_{\mu\nu}, \qquad \delta\mathcal{N} = D_y(\zeta\mathcal{N}), \tag{21}$$

$$\delta N^{\mu} = \mathcal{N}^2 g^{\mu\nu}\partial_\nu\zeta. \tag{22}$$

Let us now built the analogous of the Wilson lines (4) for gravity. Exploiting the analogy between $\bar{N}$ and A_5, we consider

$$W(y', y) = P\exp\int_y^{y'} dz\bar{N}, \tag{23}$$

or more explicitly

$$\begin{aligned} W_{y',y} = 1 &+ \int_y^{y'} dzN^{\mu}(z)\partial_\mu + \int_y^{y'} dz_1N^{\mu_1}(z_1)\partial_{\mu_1}\int_y^{z_1} dz_2N^{\mu_2}(z_2)\partial_{\mu_2} + \ldots \\ &+ \int_y^{y'} dz_1N^{\mu_1}(z_1)\partial_{\mu_1}\int_y^{z_1} dz_2N^{\mu_2}(z_2)\partial_{\mu_2}\ldots\int_y^{z_{p-1}} dz_pN^{\mu_p}(z_p)\partial_{\mu_p} + \ldots \end{aligned} \tag{24}$$

Now $W(y', y)$ defines a mapping from functions (scalar fields) on Σ_y to functions (scalar fields) on $\Sigma_{y'}$. Explicitly, let $\phi(x)$ be a scalar field defined on the hypersurface Σ_{y_0} and consider

$$\phi_y = W(y, y_0)(\phi). \tag{25}$$

Then ϕ_y verifies the equation $\partial_y \phi_y = \bar{N}_y(\phi_y)$ and is subject to the boundary condition $\phi_{y_0} = \phi$. Let $\xi(y)$ generate a y-dependent 4D diffeomorphism then from the transformation of $\bar{N}$ given in (17) we get

$$\delta W(y', y) = \xi(y')W(y', y) - W(y', y)\xi(y), \tag{26}$$

which implies that indeed ϕ_y defined in (25) transforms under diffeomorphisms as $\delta\phi_y = \xi(y)(\phi_y)$ if ϕ transforms as $\delta\phi = \xi(y_0)(\phi)$. A convenient and useful way of writing (25) is

$$\phi_y = \phi \circ X(y, y_0), \tag{27}$$

where $X(y, y_0)$ is a mapping from the manifold Σ_y to Σ_{y_0} generated by $\bar{N}$, that is

$$\partial_y X^\mu(y, y_0; x) = N_y^\mu(x), \qquad X^\mu(y_0, y_0; x) = x^\mu, \tag{28}$$

which can be written as

$$X^\mu(y, y_0; x) = W(y, y_0)(x^\mu), \tag{29}$$

$$= x^\mu + \int_{y_0}^{y} dz N^\mu(z; x) + \int_{y_0}^{y} dz_1 N^\nu(z_1; x) \int_{y_0}^{z_1} \partial_\nu(N^\mu(z_2; x)) + \dots \tag{30}$$

the right hand side of the first line being understood as the action of the W on the function x^μ.

It is possible to extend W so that it maps tensors of arbitrary rank on Σ_y to tensors on $\Sigma_{y'}$. This is done with the help of the Lie derivative as follows

$$W(y', y) = P \exp \int_y^{y'} dz L_{\bar{N}}. \tag{31}$$

It reduces to the previous expression (23) when acting on scalars. The Leibniz rule for the Lie derivative results in a simple action of W on the direct product of tensor:

$$W(y', y)(T_1 \otimes T_2) = [W(y', y)(T_1)] \otimes [W(y', y)(T_2)], \tag{32}$$

where T_1 and T_2 are arbitrary tensors on Σ_y. The commutation of the Lie derivative and the exterior derivative when acting on forms translates also to the simple property

$$d[W(y', y)(\omega_y)] = W(y', y)(d\omega_y), \tag{33}$$

where ω_y is an arbitrary form defined on Σ_y. The geometric interpretation of the map W goes as follows: when y and y' are infinitesimally close, W maps the point P with coordinates x on Σ_y to the point Q with coordinates $x^\mu + \delta y N^\mu$ on $\Sigma_{y+\delta y}$.

We are now in a position of performing the discretization of the Einstein–Hilbert action along the y-direction. We replace y by ia with i an integer and a the lattice spacing. The fields are thus the metric on each site $g^{(i)}_{\mu\nu}$, the lapse fields $\mathcal{N}^{(i)}$ and the Wilson line $W(i, i+1)$ which, as in the gauge theory, replaces the shift vector. The y derivative appears in the continuum in the combination D_y. The Lie derivative of a tensor field with respect to D_y can be written as

$$\mathcal{L}_{D_y} T_y = \lim_{\delta y \to 0} \frac{W(y, y+\delta y) T_{y+\delta y} - T_y}{\delta y}. \tag{34}$$

From this we see that the simplest discrete counterpart of the Lie derivative along D_y is

$$\Delta_{\mathcal{L}} T_i = \frac{W(i, i+1) T_{i+1} - T_i}{a}. \tag{35}$$

It is now immediate to get the discretized Einstein–Hilbert action from (8)

$$\begin{aligned} S = M^3_{(5)} a \sum_i \int d^4x \sqrt{-g_i} \mathcal{N}_i \\ \times \left[R(g_i) + \frac{1}{4\mathcal{N}_i^2} (\Delta_{\mathcal{L}} g_i)_{\mu\nu} (\Delta_{\mathcal{L}} g_i)_{\alpha\beta} \left(g_i^{\mu\nu} g_i^{\alpha\beta} - g_i^{\mu\alpha} g_i^{\nu\beta} \right) \right] \end{aligned} \tag{36}$$

The action is invariant under the product of all diffeomorphism groups associated to the points of the lattice. Under such a transformation generated by ξ_i, the different fields transform as

$$\begin{aligned} \delta g_i &= L_{\xi_i} g_i, \qquad \delta \mathcal{N}_i = \xi_i(\mathcal{N}_i), \\ \delta W(i, i+1) &= \xi_i W(i, i+1) - W(i, i+1) \xi_{i+1}. \end{aligned} \tag{37}$$

These reduce in the continuum limit to (16), (18) and (26). The explicit expression of the components of $W(i, i+1) T_{i+1}$ can be easily written down with the help of $X^\mu(i, i+1; x)$, a mapping between the manifolds at i and $i+1$ which is the discrete counterpart of $X^\mu(y, y_0; x)$ defined in (30). In fact, we have

$$[W(i, i+1) T_{i+1}]_{\mu_1, \ldots \mu_r}(x) = T_{i+1}(X^\mu(i, i+1; x))_{\nu_1, \ldots \nu_r} \partial_{\mu_1} X^{\nu_1} \ldots \partial_{\mu_r} X^{\nu_r}. \tag{38}$$

The variation of the action with respect to $W(i, i+1)$ amounts to a variation with respect to the mappings $X^\mu(i, i+1; x)$.

Notice that the action is not the most general action with the symmetries (37) since it descends from a 5D theory which had also a reparametrization invariance along the y direction. This will have very important consequences as we will now discuss.

3. SPECTRUM OF THE ACTION

We are thus led to consider *multigravity* theories such as (36). Let us first make a naive counting of the degrees of freedom (d.o.f.) that arise from a generic action with the symmetries we explicitly implemented in the action (36), with a finite number, N, of sites. We started with $N \times 10$ d.o.f. in the $4D$ metrics, N lapse fields and $(N-1) \times 4$ d.o.f. in the mappings $W(i, i+1)$. The total number of d.o.f. is thus $15 \times N - 4$. The action has local invariances with $4N$ parameters due to the 4D diffeomorphism on the N manifolds, this reduces the number of d.o.f. to $(15 \times N - 4) - (4+4)N = 7N - 4$. Out of these we expect to have one graviton (2 d.o.f.) and $N - 1$ massive spin 2 particles (5N-5 d.o.f.). The remaining degrees of freedom are expected to be shared by a number of zero modes (scalars and vectors), which does not depend on N but depends on the boundary conditions, as well as a number of massive scalars. The latter number depends on N as $2N + c$, where c is a constant which depends on the boundary conditions but which does not depend on N. These scalars are potentially pathologic, they may lead to ghosts or tachyons. For a generic multigravity theory, ghosts and instabilities do indeed appear (Aragone and Chela-Flores, 1972; Boulware and Deser, 1972; Chamseddine, 2003; Damour *et al.*, 2002, 2003; Damour and Kogan, 2002; Isham *et al.*, 1971; Isham and Storey, 1978; Salam and Strathdee, 1977). The higher dimensional theory we started with, before discretization, does not have these pathologies. It is thus possible that the action (36) inherited the consistency of the continuum action. As we will now discuss, this is indeed the case at least to the quadratic order in the fluctuations. We will show that the massive scalar modes decouple at the quadratic level. This is due to an extra local symmetry which removes $2N - 2$ degrees of freedom. For that purpose, we first perform a Weyl rescaling on the metric in (36) by defining the metrics $\gamma^{(i)}_{\mu\nu}$ and the scalars $\phi^{(i)}$ by $g^{(i)}_{\mu\nu} = \exp(-\frac{\phi_{(i)}}{\sqrt{3}})\gamma^{(i)}_{\mu\nu}, \mathcal{N}^{(i)} \equiv \exp(\phi^{(i)}/\sqrt{3})$. Then we expand the action around the vacuum

$$\gamma^{(i)}_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_p} h^{(i)}_{\mu\nu}, \qquad \phi^{(i)} = \frac{\varphi^{(i)}}{M_p}, \quad X^\mu(i, i+1) = x^\mu + \frac{a}{M_p} n^\mu_{(i)}, \quad (39)$$

and keep the quadratic fluctuations in the fields. It turns out convenient to work with a discrete Fourier transform of the fields. So for each fluctuation $\mathcal{F}_{(i)}$ with $\mathcal{F}_{(i+N)} = \mathcal{F}_{(i)}$ we define $\hat{\mathcal{F}}_{(k)}$ by

$$\hat{\mathcal{F}}_{(k)} = \sum_j \frac{1}{\sqrt{N}} \mathcal{F}_{(j)} e^{-i2\pi jk/N}. \qquad (40)$$

The discretized quadratic action becomes

$$\int d^4x \sum_k \frac{1}{4}\big\{\partial_\rho \hat{h}^{\mu\nu}_{(k)}\partial_\sigma \hat{h}^{*\alpha\beta}_{(k)}\big(\eta^{\rho\sigma}\eta_{\mu\nu}\eta_{\alpha\beta} - \eta^{\rho\sigma}\eta_{\mu\alpha}\eta_{\nu\beta} + 2\delta^{\sigma}_{(\nu}\eta_{\mu)\beta}\delta^{\rho}_{\alpha} - \eta_{\mu\nu}\delta^{\sigma}_{\beta}\delta^{\rho}_{\alpha}$$

$$- \eta_{\alpha\beta}\delta^{\sigma}_{\nu}\delta^{\rho}_{\mu}\big)\big\} - \frac{1}{2}\sum_k \partial_\mu\hat{\varphi}^{(k)}\partial_\nu\hat{\varphi}^{*(k)}\eta^{\mu\nu} - \frac{1}{4}\big(\partial_\mu \hat{n}^{(0)}_\nu - \partial_\nu \hat{n}^{(0)}_\mu\big)\big(\partial^\mu \hat{n}^{\nu(0)} - \partial^\nu \hat{n}^{\mu}_{(0)}\big)$$

$$+ \sum_{k\neq 0}\frac{1}{a^2}\sin^2\frac{\pi k}{N}\left\{\left(\left(\hat{h}^{(k)}_{\mu\nu} - \frac{\eta_{\mu\nu}}{\sqrt{3}}\hat{\varphi}^{(k)}\right) - \frac{2a\partial_{(\mu}\hat{n}^{(k)}_{\nu)}}{e^{i2\pi k/N}-1}\right)\left(\left(\hat{h}^{*(k)}_{\alpha\beta} - \frac{\eta_{\alpha\beta}}{\sqrt{3}}\hat{\varphi}^{*(k)}\right)\right.\right.$$

$$\left.\left. - \frac{2a\partial_{(\alpha}\hat{n}^{*(k)}_{\beta)}}{e^{-i2\pi k/N}-1}\right)(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta})\right\}. \tag{41}$$

The spin two and one content of the action is easily read from the action. We have one massless spin 2 particle given by $\hat{h}^{(0)}_{\mu\nu}$, one massless spin 1 particle $\hat{n}^{(0)}_\mu$, one massless scalar $\hat{\phi}^{(0)}$ and a tower of massive spin two particles with a spectrum given by

$$m_k^2 = \frac{1}{a^2}\sin^2\frac{\pi k}{N}. \tag{42}$$

The action has the local invariances

$$\delta\hat{h}^{(k)}_{\mu\nu} = 2\partial_{(\mu}\xi^{(k)}_{\nu)}, \qquad \delta\hat{n}^{(k)}_\mu = \frac{(e^{i2\pi k/N}-1)}{a}\xi^{(k)}_\mu, \tag{43}$$

which show that for $k \neq 0$, the $\hat{n}^{(k)}_\mu$ are Stuckelberg fields which are absorbed by the massive spin 2 fields and do not propagate. The invariances (43) are the linearized version of the invariance under 4D diffeomorphisms, they are expected by construction. Less expected is the invariance under the local transformations

$$\delta\hat{h}^{(k)}_{\mu\nu} = \eta_{\mu\nu}f^{(k)}, \quad \delta\hat{\varphi}^{(k)} = \sqrt{3}f^{(k)}, \quad \delta\hat{n}^{(k)}_\mu = \frac{a}{1-e^{-i2\pi k/N}}\partial_\mu f^{(k)}, \quad k\neq 0. \tag{44}$$

A generic multigravity theory with 4D diffeomorphism invariance on each site realized does not possess this symmetry, which is inherited from the diffeomorphism invariance under the y reparametrizations in the continuum theory. In fact the invariance under (44) eliminates, at the quadratic level, all except the massless, scalar modes $\hat{\phi}^{(k)}$. It may also be used to eliminate the trace of the $\hat{h}^{(k)}_{\mu\nu}$ proving the absence of ghostlike excitations. Associated to this local invariance there is a constraint which removes one more set of scalars. At this point we note that while the Pauli–Fierz action removes the ghost by hand, the above action removes it with the aid of a local symmetry; in the gauge where $\hat{\varphi}^{(k)}$ and $\hat{n}^{(k)}_\mu$ are zero the action reduces to the Pauli–Fierz form with no extra propagating scalar. This can be seen more explicitly by considering the scalar modes separately.

An open question is whether this symmetry persists at cubic or higher level. This seems unlikely but deserves further investigation.

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